Tangible: What a Cut-Up!

Equipment: graph paper, scissors, ruler, protractor

Procedure:

- Draw a parallelogram defined by the vectors $\vec{A} = \langle 7,16,0 \rangle$ cm and $\vec{B} = \langle -10,1,0 \rangle$ cm. (Note: the graph paper lines are not a cm apart, and this time, I want you to measure out 7 cm, 16 cm, etc.) Label the vectors on the parallelogram and write "TOP" on the top face of the parallelogram BEFORE CUTTING. Cut the parallelogram out.
- Find the area of the parallelogram. Write it somewhere on the face.
- Find the lengths of the edges of the parallelogram, and the angle between $\vec{A}$ and $\vec{B}$. Write them on the face also.
- Multiply the lengths of the two edges by the sine of the angle between $\vec{A}$ and $\vec{B}$. Is this number the same as the area you found earlier?
- Use the right-hand rule – place your fingers along $\vec{A}$ and curl them into $\vec{B}$. Label the top of the parallelogram with the direction of the vector found by the RHR.
- Finally, mathematically compute the cross product using $\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$. Does the area agree with the magnitude of this vector? Does the direction agree? The units?

This is what it means to cross two vectors: we're finding the area of a parallelogram spanned by those two vectors, with the direction indicated by the RHR – this concept of an area with a direction will come up again in E&M next semester.

- Flip the parallelogram over (now you're not looking at TOP – you're looking at BOTTOM). How has the situation changed? Line $\vec{B}$ up with the hole in the paper, and determine your new vector $\vec{A}$.
- What is the magnitude of $\vec{A} \times \vec{B}$? What is the direction? Try the right-hand rule now.
- It is as if a vector shoots through the parallelogram in one direction, regardless of how you move it around. Whenever you have two vectors, the cross product is directed by the RHR.